

# Mass Transfer in Semifluidized Beds For Solid-Liquid System

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A new fluid-solid contacting technique known as *Semifluidization* has been studied. Mass transfer data were obtained for a benzoic acid-water system, and correlations in terms of mass transfer factors and over-all void fractions (for both packed and fluidized sections) are presented. Previous mass transfer correlations for fixed and fluidized beds are critically evaluated with respect to the limitation and applicability and are discussed in relation to semifluidized-bed mass transfer correlations.

Semifluidization is a type of fluid-solid contact operation in which the expansion of a fluidized bed is partially restricted. Under such conditions the porosity of a semifluidized bed is intermediate between the rigidly packed bed and the completely fluidized bed.

Some of the characteristics of a semifluidized bed, such as the simultaneous formation of a packed section and a fluidized section in the bed, were reported in a previous communication (9). The purpose of this paper is to report the results of experimental work on the mass transfer aspect in a semifluidized bed of a solid-liquid system.

Aside from its practical significance in mass transfer operations, the study of rate of mass transfer is of general significance because it often leads to better understanding of other aspects of chemical engineering operations and heat transfer.

Since the mechanism of interphase mass transfer in a fluidized bed is already complicated (4, 24), no particular attempts were made to analyze the data in the light of the rigorous theories of interphase mass transfer. Rather, emphasis was placed on the empirical

correlation of over-all mass transfer coefficients and the comparison of the correlations obtained with those of the fluidized and packed beds.

The relationships among the over-all mass transfer coefficients, the mass transfer coefficients in each individual bed, and the internal structure of the bed (that is, the relative size of the packed and fluidized sections in the bed) were carefully considered.

## EXPERIMENTAL

The experimental apparatus consisted of a main test column in which granular benzoic acid particles were dissolved by a measured stream of upward flowing water. A container was provided for retention of samples of each run for analysis. Figure 1 shows the diagram of the apparatus.

Water was supplied from a 50-gal. constant-head tank (A) and was forced through the system by a centrifugal pump (B). The water was initially directed through the globe valve and a flowmeter (C) as a means of controlling and measuring the rate of flow. The test column consisted of two parts, a calming section (D) and a test section (F). The test section was a vertical Pyrex tube 2 in. I.D. and 24 in. high. Two brass plates clamping a filter cloth (E) were placed between the calming and the test sections to establish an even distribution of the upward flow. A rigid close-fitting sieve plate (G) was attached to a long rod which could be moved up or down to a fixed position. This was installed in the test column to adjust bed

depth and prevent the complete expansion of the solid bed.

For the second series of experiments the apparatus was modified.

In the first series only the inlet and outlet concentrations were measured, whereas in the second series the axial concentration gradient of the fluid was measured. Since the samples were obtained from inlet and outlet pipes in the first series of experiments, they could be considered as the average samples across the cross section of the column. The sampling lines having 1/4-in. I.D. were employed in the second series of experiments. They were sufficiently large to assume withdrawal of uniform average samples across the cross section of the column, as evidenced by the fact that the concentration of outlet samples obtained from the sampling line just above the top of the screen was almost identical with those obtained from the outlet of the column. The apparatus used in the latter series was the same as for the former series, except the test section. This new test section was made from a 2-in. I.D. standard iron pipe with cocks attached for sampling, at 2-in. spacings. A sheet of filter cloth was cemented to the inside wall over the outlet of each cock to prevent the plugging of the solid benzoic acid particles from the sampling lines. The sizes of particles used (8 ~ 24 mesh) were much larger than the opening of the filter cloths, and the particles were freed from the dust by a washing process suggested by Evans and Gerald (8). No escaping of the particles from the sampling cocks was observed.

Although the solubility of the benzoic acid in water is low, in order to minimize the change of geometric shape of the particles during the experimental runs the individual runs were not permitted to exceed an 80-sec. duration. A decrease of less than 3% of the initial weight of the solid particles for a single run was experienced. Intermittent analysis of effluent liquid concentration during the run indicates that steady state operation has been achieved.

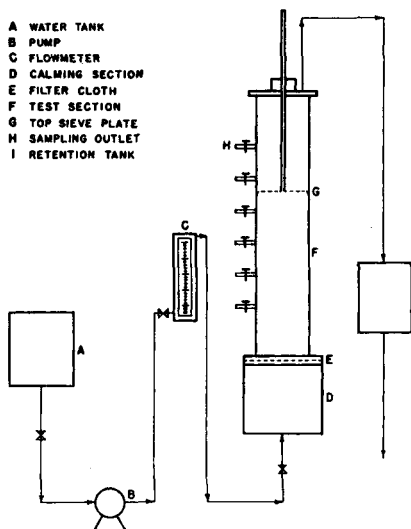


Fig. 1. Diagram of experimental apparatus.

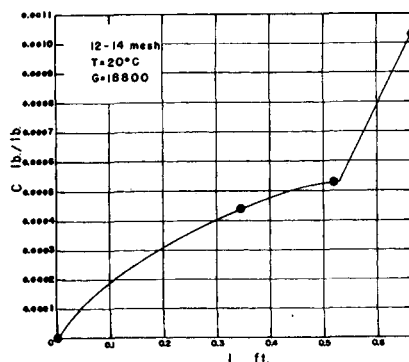


Fig. 2. Axial concentration profile.

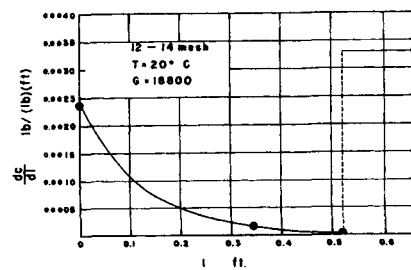


Fig. 3. Axial concentration gradient.

TABLE 1. CALCULATION OF FIRST SERIES OF SEMIFLUIDIZED DATA (REPRESENTATIVE RUNS)

Run no.	$T, ^\circ\text{C.}$	$W, \text{lb.}$	$h_o, \text{ft.}$	$h, \text{ft.}$	$h_r, \text{ft.}$
A-1	22.3	0.4916	0.5888	0.8091	0.4008
B-1	21.00	0.2533	0.2999	0.5008	0.3342
C-1	22.60	0.2645	0.3093	0.4675	0.2550

Run no.	$h_{po}, \text{ft.}$	$X$	$R$	$\epsilon(\text{over-all})$	$D_p, \text{ft.}$	$A, \text{sq. ft.}$
A-1	0.4083	0.6935	1.3732	0.6548	0.00690	10.40
B-1	0.1666	0.5555	1.6700	0.7129	0.004531	7.62
C-1	0.2125	0.6873	1.5114	0.6787	0.002610	12.50

Run no.	$G, \text{lb.}/(\text{hr.})$ (sq. ft.)	$G', \text{lb.}/\text{hr.}$	$C_s, \text{lb.}/\text{lb.}$	$C_b, \text{lb.}/\text{lb.}$	$k_{im}, \text{lb.}/(\text{hr.})$ (sq. ft.) ( $\Delta C$ )	$N_{sc}$
A-1	16,067	350.52	0.003140	0.001640	24.90	1,120
B-1	10,610	231.47	0.003005	0.001563	22.30	1,220
C-1	4,500	98.17	0.003177	0.003051	25.38	1,110

Run no.	$N_{sh}$	$N'_{Re}$	$N'_{Re}/(1-\epsilon)$	$J_d$	$(N_{sh} \epsilon - 2)/N \ 1/3 \ sc$	$J'_d$
A-1	64.48	48.33	140.0	0.1670	3.87	0.1245
B-1	39.74	20.25	70.53	0.2400	2.46	0.1745
C-1	24.63	5.138	15.99	0.6045	1.61	0.4255

TABLE 2. CALCULATION OF SECOND SERIES OF SEMIFLUIDIZED DATA (REPRESENTATIVE RUN)

Run no.	$W, \text{lb.}$	$h_o, \text{ft.}$	$h, \text{ft.}$	$h_r, \text{ft.}$
D-1	0.3307	0.3613	0.5717	0.3484

Run no.	$h_{po}, \text{ft.}$	$X$	$D_p, \text{ft.}$	$A, \text{sq. ft.}$	$\epsilon$	$S, \text{sq. ft.}$
D-1	0.2233	0.6182	0.006890	6.97	0.7004	0.023916

Run no.	Height of sampling outlet, ft.			$C, \text{lb.}/\text{lb.}$ at			$C_1, \text{lb.}/\text{lb.}$	$C_2, \text{lb.}/\text{lb.}$
	$a$	$b$	$c$	$a$	$b$	$c$		
D-1	0.2033	0.3700	0.5717	0.000215	0.000359	0.000943	0.00030	0.000943

Run no.	$C_s, \text{lb.}/\text{lb.}$	Packed bed	$(\Delta C)_{im}$ Fluidized bed	Over-all	Packed bed	$(\Delta C)_{im}$ Fluidized bed	Over-all
D-1	0.002902	0.002271	0.002752	0.002405	0.002380	0.002752	0.002460

Run no.	$G, \text{lb.}/(\text{hr.})$ (sq. ft.)	$G', \text{lb.}/\text{hr.}$	Packed bed	Fluidized bed	Over-all	Packed bed	Fluidized bed	Over-all
D-1	21,524	51,477	30.83	21.08	28.97	28.68	21.08	28.32

Run no.	$N'_{Re}$	$N'_{Re}/(1-\epsilon)$	$N_{sh}$	$J_d$
D-1	60.88	203.30	81.00	79.3

Run no.	$N_{sc}$	$(N_{sh} \epsilon - 2)/N \ 1/3 \ sc$	$J_d$
D-1	1,290	5.03	4.91

All the experiments were carried out at room temperature, and the fluid temperature was measured at the inlet and outlet of the column. The determination of the concentration of the water sample was made by direct titration with 0.04 N.

sodium hydroxide with phenolphthalein indicator. No difficulty was encountered in securing reproducible end points. The acid concentration varied from 0.0004 g. acid/g. water to 0.0030 g. acid/g. water. Granular particles of benzoic acid

employed in the present investigation were prepared in three screen ranges of 8 to 10 mesh, 12 to 14 mesh, and 20 to 24 mesh by the method of Evans and Gerald (8). Several tests indicated that the reproducibility of the particle characteristics by the Evans and Gerald method was excellent with deviation of less than  $\pm 1\%$  error. Significant characteristics of the particles are summarized in the Evans and Gerald paper (8).

Solution saturated with the benzoic acid used in the present investigation was found by titration to be in agreement with the literature value (22). Also this solubility can be represented by an equation reported by Ishine (16) as a function of temperature:

$$C_s = 0.17 + 0.00197t + 0.00020t^2 \quad (t: 0^\circ \text{ to } 30^\circ\text{C.}) \quad (1)$$

This value was eventually used in this investigation.

The diffusivity and Schmidt number of the benzoic acid-water system were taken directly from the data of Chang (2), quoted by Linton and Sherwood (19). Values of the Schmidt number at four temperature levels were also reported by them (2, 19).

Since the concentrations of benzoic acid solution obtained in this work were very low, viscosity and density of water were employed to calculate the Reynolds numbers (20).

## RANGE OF EXPERIMENT

In the first series of experiments measurements included the following variables: exit concentration, water temperature, water flow rate, and bed depth. Computed variables derived from these measurements included  $k$ ,  $J_d$ ,  $N'_{Re}$ ,  $N_{sh}$ ,  $N_{sc}$ , and  $J'_d$ . The ranges of major variables were as follows:

Particle diameter: 8 to 10 mesh, 12 to 14 mesh, and 20 to 24 mesh.

Water temperature:  $16.8^\circ$  to  $24^\circ\text{C.}$

Over-all bed depth: 3 to 6 in.

Voidage: 0.65 to 0.90.

Modified Reynolds number ( $N'_{Re}$ ): 5 to 130.

Sherwood number ( $N_{sh}$ ): 21 to 116.

Schmidt number ( $N_{sc}$ ): 1,020 to 1,540.

The data and calculated results of some representative runs from the first series of experiments are tabulated in Table 1.\*

Table 2\* contains the measured and calculated data of representative runs for the second series of experiments in which the axial concentration profile of fluid was studied in addition to those variables mentioned above. A typical axial concentration profile and concen-

\* Tabular material has been deposited as document 6347 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for \$2.50 for photoprints or \$1.75 for 35-mm. microfilm.

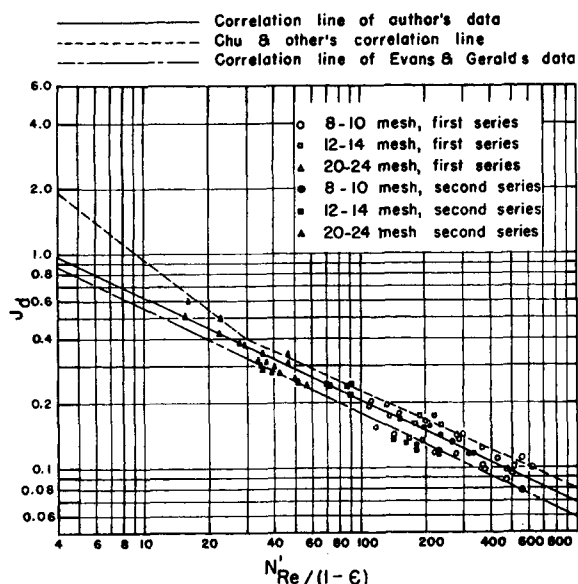


Fig. 4.  $J_d$  factor vs.  $N'_{Re}/(1-\epsilon)$  correlation of the experimental data.

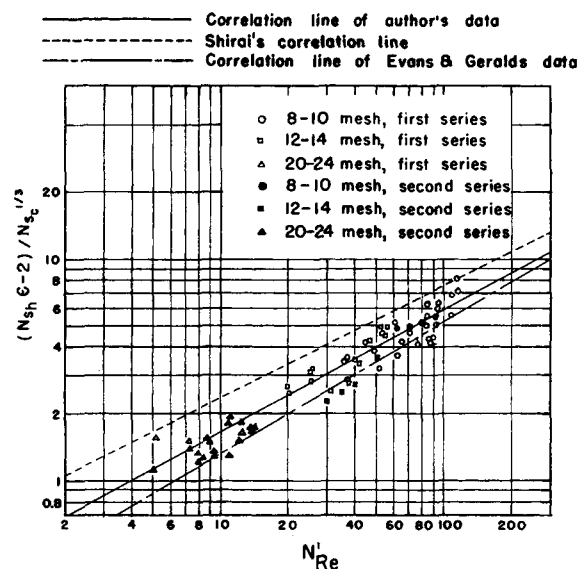


Fig. 5.  $(N_{Sh} \epsilon - 2)/N_{Sc}^{1/3}$  vs.  $N'_{Re}$  correlation of the experimental data.

tration gradient curves obtained in the second series of experiments are illustrated in Figures 2 and 3. The break point at 0.52 ft. of bed height corresponds to the boundary of fluidized bed section and fixed bed section at which a distinct discontinuity of concentration gradient exists.

#### ANALYSIS AND CORRELATION OF RESULTS

To derive the rate equation for the mass transfer between the solid particles and fluid in the semifluidized bed the following assumptions were made:

1. The concentration of the fluid in contact with the particle surface is equal to the saturated concentration.
  2. The velocity distribution and concentration of fluid are uniform at any cross section of the test section.
  3. The surface area of the particles per unit volume in each bed is uniform.
  4. Axial turbulent diffusion coefficient,  $E_z$ , in each bed remains constant under definite flow conditions.
  5. The process is isothermal.
- The following material balance could be obtained for any bed height:

$$G \frac{dC}{dl} = E_z \rho_f \frac{d^2C}{dl^2} + k a (C_s - C) \quad (2)$$

When one assumes that longitudinal or axial mixing in the bed can be negligible

$$E_z \rho_f \frac{d^2C}{dl^2} = 0 \quad (3)$$

Then Equation (2) can be expressed as (1, 5)

$$G \frac{dC}{dl} = k a (C_s - C) \quad (4)$$

Since there exists a finite discontinuity in the axial concentration gradient at the interphase of the fluidized and packed beds,

$$G \int_{c_0}^{c_1} \frac{dC}{C_s - C} + G \int_{c_1}^{c_2} \frac{dC}{C_s - C} = (k_f)_{1m} a_f \int_0^{l_1} dl + (k_{pa})_{1m} a_{pa} \int_{l_1}^{l_2} dl \quad (5)$$

or

$$G \ln \frac{C_s - C_0}{C_s - C_1} + G \ln \frac{C_s - C_1}{C_s - C_2} = (k_f)_{1m} a_f h_f + (k_{pa})_{1m} a_{pa} h_{pa} \quad (6)$$

But Equation (6) can be further simplified:

$$G \ln \frac{C_s - C_0}{C_s - C_2} = (k_f)_{1m} A_f + (k_{pa})_{1m} A_{pa} \quad (7)$$

If the weighted mean value of over-all mass transfer coefficient is defined as

$$k_{1m} = (k_f)_{1m} \frac{A_f}{A} + (k_{pa})_{1m} \frac{A_{pa}}{A} \quad (8)$$

or

$$k_{1m} = (1 - X) (k_f)_{1m} + X (k_{pa})_{1m}$$

Equation (7) may be transformed into

$$G \ln \frac{C_s - C_0}{C_s - C_2} = k_{1m} A \quad (9)$$

or

$$k_{1m} = \frac{G' (C_2 - C_0)}{A (\Delta C)_{1m}} \quad (10)$$

In order to account for the effect of axial mixing, which was neglected,  $k_{1m}$  must include a correction factor  $F$  suggested by Epstein (7, 13).

In case the axial concentration data are available, as in the second series of experiments, the value of  $k$  in each bed can be directly calculated by use of the integrated mean driving force as

$$(k_f)_{1m} = \frac{G' (C_1 - C_0)}{A_f (\Delta C_f)_{1m}} \quad (11)$$

and

$$(k_{pa})_{1m} = \frac{G' (C_2 - C_1)}{A_{pa} (\Delta C_{pa})_{1m}} \quad (12)$$

The over-all mass transfer coefficient based on integrated mean driving force may be evaluated as

$$k_{tm} = \frac{G' (C_2 - C_0)}{A (\Delta C)_{tm}} = \frac{G' (C_1 - C_0) + G' (C_2 - C_1)}{A (\Delta C)_{tm}} = \frac{(k_f)_{1m} A_f (\Delta C_f)_{1m} + (k_{pa})_{1m} A_{pa} (\Delta C_{pa})_{1m}}{A (\Delta C)_{tm}} \quad (13)$$

Equation (13) implies that

$$k_{tm} = (1 - X) \frac{(\Delta C_f)_{tm}}{(\Delta C)_{tm}} (k_f)_{tm} + X \frac{(\Delta C_{pa})_{tm}}{(\Delta C)_{tm}} (k_{pa})_{tm} \quad (14)$$

First, the mass transfer data were correlated in terms of the  $J_d$  factor and modified Reynolds number as is illustrated in Figure 4 (3, 14). The mass transfer coefficients were calculated on the basis of the over-all logarithmic mean driving force, as was done by other investigators (3, 8) for the packed- and fluidized-bed mass transfer. The effect of natural convection on the rate of mass transfer which would become appreciable for the large particles ( $D_p > 1/4$  in.) at the very low Reynolds numbers as indicated by Gaffney (12) was neglected, and no attempt was made to introduce Grashof number in the correlation of data. The broken line in Figure 4 represents the generalized correlation used by Chu et al. (4) to correlate the mass transfer data for both packed and fluidized beds. The solid line correlated all the semifluidization data with the magnitude of the deviations comparable to the correlation used by Chu et al. (4). The equation for the solid line was found to be

$$J_d = 1.865 \left( \frac{N'_{Re}}{1 - \epsilon} \right)^{-0.480} \quad (15)$$

Shirai (23) presented the following correlation of the mass transfer data in the particle beds as suggested by the boundary-layer theory (10, 16):

$$N_{Sh}\epsilon = 2.0 + 0.75 N'_{Re}{}^{1/2} N_{Sc}{}^{1/3} \quad (16)$$

Figure 5 shows the similar correlation for the present data and a comparison with Shirai's equation (23). The regression line through the data is expressed as

$$N_{Sh}\epsilon = 2.0 + 0.46 N'_{Re}{}^{0.548} N_{Sc}{}^{0.38} \quad (17)$$

To take the effect of molecular diffusion and natural convection into account the  $J_d$  factor can be modified as (23)

$$J'_d = \frac{N_{Sh} - 2}{(N'_{Re})(N_{Sc})^{1/3}} \quad (18)$$

The  $J'_d$  factors are also correlated as a function of  $N'_{Re}/(1 - \epsilon)$  in Figure 6, which yield the relationship

$$J'_d = 1.51 \left( \frac{N'_{Re}}{1 - \epsilon} \right)^{-1/2}$$

or

$$N_{Sh} = 2.0 + 1.51[(1 - \epsilon)N'_{Re}]^{1/2} (N_{Sc})^{1/3} \quad (19)$$

The data obtained in the second series of experiments are also similarly correlated, and the results are plotted on Figures 4 to 6 again for comparison with the results of the first series of experiments.

## DISCUSSION AND CONCLUSION

In order to evaluate the effect of the packed-bed formation on the mass transfer in the fluidized bed, the mechanism of the packed-bed formation should be understood. The size of the packed-bed section formed must be estimated by an equation including the independent variables of the experiment and the characteristics of ordinary fluidization under similar flow conditions. Such an equation was already reported by the authors in the previous communication (9).

It was also reported in the communication that the over-all mass transfer coefficients based on the over-all logarithmic driving forces can be correlated by use of  $J_d$  factor suggested by Chu and his co-workers (3) within the range of maximum probable deviation. Referring to Figure 4, one can see that Chu's original  $J_d$  factor correlation yields  $J_d$  values consistently higher than the present experimental data. Chu's correlation pertains only to the ordinary fluidized and packed beds. The maximum deviation of the present data based on his correlation appears to be 22% in the range of Reynolds numbers used. Evans and Gerald's (8) data for the benzoic acid-water system were not included in the original correlation by Chu et al. (3). They were recalculated in terms of  $J_d$  vs.  $N'_{Re}/$

$(1 - \epsilon)$ . It clearly indicates that Evans and Gerald's (8) data also deviate consistently from the original correlation of Chu et al. (3) with approximately the same magnitudes as those of the semifluidized bed data. Their packed bed data are correlated as

$$J_d = 2.132 \left( \frac{N'_{Re}}{1 - \epsilon} \right)^{-0.512} \quad (20)$$

and the fluidized bed data are correlated as

$$J_d = 1.340 \left( \frac{N'_{Re}}{1 - \epsilon} \right)^{-0.468} \quad (21)$$

The over-all correlation of Evans and Gerald's (8) data may be expressed as

$$J_d = 1.680 \left( \frac{N'_{Re}}{1 - \epsilon} \right)^{-0.490} \quad (22)$$

Comparison of Equations (22) with (15) reveals that within the range of the variables of the present investigation and the errors of experimentation the semifluidization data may be correlated as well as the data for ordinary fluidized and packed beds by use of the  $J_d$  factor vs.  $N'_{Re}/(1 - \epsilon)$  plot based on the over-all logarithmic mean mass transfer coefficient,  $k_{lm}$ .

The data for the benzoic acid-water system were not included in Shirai's (23) generalized correlation covering the data of the fluidized bed, fixed bed, and a single particle. Evans and Gerald's (8) data were again recorelated by the method suggested by Shirai (23) as illustrated in Figure 5, which indicates that the experimental data were consistently lower than the

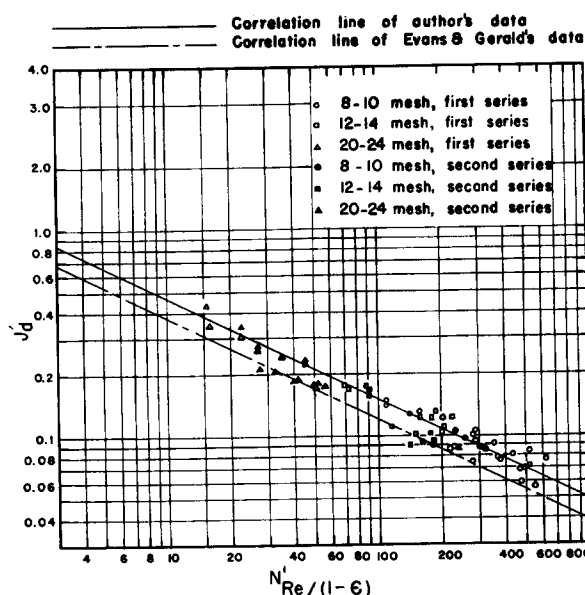


Fig. 6.  $J'_d$  factor vs.  $N'_{Re}/(1 - \epsilon)$  correlation of the experimental data.

calculated values by approximately 38%. This is, however, within the range of accuracy expected from the generalized correlation of Shirai (23) for any specific system. Evans and Gerald's data are expressed as follows:

For packed bed:

$$N_{sh}\epsilon = 2.0 + 0.424 N'_{Re}{}^{0.558} N'_{Sc}{}^{1/3} \quad (23)$$

For fluidized bed:

$$N_{sh}\epsilon = 2.0 + 0.230 N'_{Re}{}^{0.659} N'_{Sc}{}^{1/3} \quad (24)$$

For over-all correlation:

$$N_{sh}\epsilon = 2.0 + 0.312 N'_{Re}{}^{0.608} N'_{Sc}{}^{1/3} \quad (25)$$

The over-all correlation expressed by Equation (25) is again very similar to that for the semifluidized-bed data expressed by Equation (16) and Figure 5. These similarities of the correlation between the ordinary packed beds and semifluidized beds by use of the methods suggested by Chu et al. (3) and Shirai (23) would indicate that it is possible, at least for the purpose of correlation, to consider the semifluidized bed to possess uniform solid-density distribution. When correlations are not available for any specific system for semifluidization, the generalized mass transfer correlation of the ordinary packed and fluidized beds may be employed to estimate the mean mass transfer coefficients.

Equation (19), which was derived by modification of the  $J_d$  factor correlation to the present data, yields a mean deviation of 11.7%; Equations (15) and (17) give a deviation of 9.87% and 11.73% respectively. This implies that the use of the modified correlation as expressed by Equation (19) does not improve the correlation; however, the form of this equation would agree with the expression predicted by the boundary-layer theory (16).

Evans and Gerald's (8) data were recorelated in the form of Equation (19). The packed-bed data are correlated as

$$N_{sh} = 2.0 + 1.483[(1 - \epsilon) N'_{Re}]^{1/2} (N'_{Sc})^{1/3} \quad (26)$$

and the fluidized-bed data are correlated as

$$N_{sh} = 2.0 + 1.031[(1 - \epsilon) N'_{Re}]^{1/2} (N'_{Sc})^{1/3} \quad (27)$$

An over-all correlation equation may be expressed as

$$N_{sh} = 2.0 + 1.202[(1 - \epsilon) N'_{Re}]^{1/2} (N'_{Sc})^{1/3} \quad (28)$$

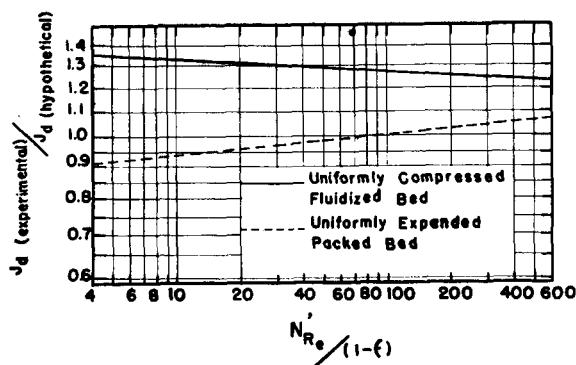


Fig. 7. Comparison of experimental value of mass transfer factor with those of hypothetical models.

In spite of the similarity and analogy as described above in correlations of the mass transfer data in the fluidized, semifluidized, and packed beds, a semifluidized bed may be considered neither a uniformly compressed fluidized bed nor a uniformly expanded packed bed as illustrated in Figure 7. The lines in Figure 7 correspond to the ratios of Equation (15) with Equations (20) and (21) respectively. The over-all mass transfer coefficients in the semifluidized bed appear greater than those of the hypothetical uniformly compressed fluidized bed with the same over-all bed porosity and smaller than those of the hypothetical uniformly expanded packed bed. As can be seen from the formation of the distinct sections of the fluidized and packed beds, which are aligned in series, the over-all mass transfer coefficients in the semifluidized beds would be expected to be a weighted average between  $k_f$  and  $k_{pa}$  obtained under the same flow conditions. This fact is expressed as Equations (8) and (14). Mass transfer coefficients calculated from these equations gave the mean deviations from the experimental values of -0.78 and -4.3% respectively. Since (9)

$$X = \frac{\rho_s S(1 - \epsilon_{pa})(h_{fo} - h)}{W} \frac{(1 - \epsilon_f)}{(\epsilon_f - \epsilon_{pa})} \quad (29)$$

Equation (8) can be written as

$$k_{1m} = \left[ 1 - \frac{\rho_s S(1 - \epsilon_{pa})(h_{fo} - h)}{W} \frac{(1 - \epsilon_f)}{(\epsilon_f - \epsilon_{pa})} \right] (k_f)_{1m} + \frac{\rho_s S(1 - \epsilon_{pa})(h_{fo} - h)}{W} \frac{(1 - \epsilon_f)}{(\epsilon_f - \epsilon_{pa})} (k_{pa})_{1m} \quad (30)$$

An equation similar to Equation (30) to estimate  $k_{tm}$  can be derived by a

combination of Equations (14) and (30). Equation (30) enables one to estimate the over-all mass transfer coefficients from the knowledge of ordinary packed and fluidized beds operated under the same conditions. By varying  $X$ , which in turn depends on the over-all porosity of the bed and can be arbitrarily fixed for the semifluidized bed, one should be able to obtain any value of  $k$  between  $k_f$  and  $k_{pa}$ . Since  $1 - \epsilon = (1 - \epsilon_o)/R$ , Equation (15) or (19) indicates that approximately such a change of over-all  $k$  can be controlled as inversely proportional to  $\sqrt{R}$ . This could be illustrated by use of the data from the experimental runs at  $G = 28,000$  lb./ (hr.) (sq.ft.) with the mean deviation of 7%. It may be desirable to point out again that under definite flow conditions the values of  $k_f$  for fluidized and  $k_{pa}$  for packed beds, unlike the corresponding  $k$  in the semifluidized bed, are fixed for any particular system.

In the first series of experiments the axial concentration profile was not determined, and the logarithmic driving force was made use of in the correlation of the data. It has already been pointed out that the  $k_{1m}$  must contain in itself the  $F$  factor of Epstein (7). The importance of the axial mixing in the solid-liquid contact process has been emphasized by various investigators recently (7, 26). However it appears that so far there is no way to predict exactly the axial mixing coefficient of a particular system under certain operating conditions with sufficient accuracy for mass transfer calculations. But there is good indication that axial mixing in the liquid-solid systems is much less extensive than in the gas-solid systems (21, 26). In Figures 4, 5, and 6 the mass transfer coefficients calculated by use of the integrated mean driving force are compared with the correlation obtained in the first series of experiments. They indicate that the deviation of the second series experimental data from the correlation are within the range of the first series of experiments. This implies that the

$F$  factor included in the correlation of the data in the first series of the experiments were close to unity; in other words the effect of axial concentration gradient change and fluid mixing were small. Since an over-all mass transfer based on integrated mean concentration difference is employed, the radial mixing effect is not considered here. The maximum deviation of the values of  $F$  from unity for the second series of experiments was 7.65%.

As indicated in the introduction, probably no theories of interphase mass transfer based on the simple models can be adopted in the correlation of mass transfer data in the packed or fluidized bed. Danckwerts (4) in his review paper suggested the necessity of the theory different from either the boundary-layer theory or the surface-renewal theory to explain the behavior of gas absorption by liquid in the packed bed. The results of mass transfer investigation by Johnson and Huang (17) using a series of organic acids including benzoic acid suggested the applicability of the surface-renewal theory to the interphase mass transfer between solid and liquid. They too however indicated the difficulty of applying the surface-renewal theory directly to the mass transfer in packed beds because of the variation of the local conditions in the packed tower. A recent study by Bennett and Lewis (1) indicated that the direct applicability of the surface-renewal theory to the interphase mass transfer between the benzoic acid and water is doubtful even in the rotating-cylinder apparatus with very simple geometry. Thus further complication in application of theories to the mass transfer in semi-fluidized bed can be easily expected because of its complex structure. As Equation (19) indicates,  $1/3$  power of Schmidt numbers has been used in the present study (with a mean deviation of 11.7%), which is, as described previously, based on the boundary-layer theory (11, 16). Recent theory proposed by Toor and Marchello (24) would predict that the power for  $N_{sc}$  should vary depending on the system and local flow conditions. It should be pointed out that in view of the relatively small range of Schmidt numbers covered in the experiments, extensive extrapolation of the empirical correlations presented in this paper must be approached with caution.

The possibility of the practical application of semifluidization has been described in the previous communication (9).

#### ACKNOWLEDGMENT

The authors express their appreciation to Henry T. Ward, Head of the Chemical Engineering Department, Kansas State

University, for his advice and encouragement and also to the Kansas State University Engineering Experiment Station for financial support.

#### NOTATION

$A$	= total particle surface area, sq. ft.
$C$	= concentration of benzoic acid in water stream, lb./lb.
$C_s$	= concentration of benzoic acid in saturated solution, lb./lb.
$dl$	= differential bed height, ft.
$D_p$	= equivalent spherical particle diameter, ft.
$D_e$	= diffusivity of benzoic acid in water, (sq. ft.)/hr.
$E_z$	= longitudinal mixing coefficient, (sq. ft.)/hr.
$F$	= correction factor, dimensionless
$f$	= function of
$G$	= superficial mass velocity, lb./hr. (sq. ft.)
$G'$	= mass velocity, lb./hr.
$h$	= height of the top sieve plate or over-all depth of bed, ft.
$h_o$	= depth of height of initial bed, ft.
$h_f$	= height of fluidized bed section, ft.
$h_{po}$	= height of packed bed section, ft.
$J_d$	= mass transfer factor, dimensionless, $[(k/G) N_{sc}^{2/3}]$
$J'_d$	= modified mass transfer factor, dimensionless
$k$	= mass transfer coefficient, lb./hr. (sq. ft.) (unit $C$ )
$k_p$	= mass transfer coefficient, ft./hr.
$l$	= distance in axial direction of bed, ft.
$N'_{Re}$	= modified Reynolds number, dimensionless, $(D_p G/\mu)$
$N_{sc}$	= Schmidt number, dimensionless
$N_{sh}$	= Sherwood number, dimensionless
$R$	= bed expansion rate, dimensionless
$S$	= cross-section area of the column, sq. ft.
$T$	= temperature, °C.
$V$	= over-all volume of the bed, cu. ft.
$W$	= weight of the particle bed, lb.
$X$	= weight fraction of particles in the packed bed section, dimensionless

#### Greek Letters

$\epsilon$	= void fraction or porosity in the bed, dimensionless
$\rho$	= fluid density, lb./cu. ft.
$\rho_s$	= bulk density of the solid, lb./cu. ft.
$\rho_s$	= solid density, lb./cu. ft.
$\mu$	= viscosity, lb./hr. (ft.)

#### Subscript

$f_o$	= completely fluidized bed or ordinary fluidized bed
$f$	= fluidized bed
$p$	= particle
$pa$	= packed bed
$0$	= at bed inlet
$1$	= at interface between packed and fluidized beds
$2$	= at bed outlet
$m$	= mean value
$lm$	= logarithmic mean value
$tm$	= integrated mean value

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Manuscript received August 13, 1959; revision received December 7, 1959; paper accepted December 16, 1959. Paper presented at A.I.Ch.E. Atlanta meeting.